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Combustion Timing Control Based on First Modal Coefficients of Individual Cylinder Pressure Traces

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Abstract

When an SI engine is equipped with individual cylinder pressure transducers, combustion timing of each cylinder can be precisely controlled by adjusting spark timing in real-time. In this paper, a novel method based on principal component analysis (PCA) is introduced to control the combustion timing with a significantly less computational burden than a conventional method.

Introduction

In an internal combustion engine, maintaining combustion timing at a pre-determined optimal value is critical to achieve the best fuel economy and combustion stability. To precisely control the combustion timing and other key engine variables, in-cylinder pressure transducer can be implemented at individual cylinders [1, 2, 3, 4, 5].

In general, combustion timing is indicated by CA50, which is the timing in crank angle when 50% of fuel mass is burned, or equivalently, 50% of heat from combustion is generated. To calculate the CA50 in real-time, it is required to perform a heat release analysis at each engine event based on measured cylinder pressure trace. The heat generated from combustion can be calculated using the following equation [6],

$$\frac{dQ_c}{d\theta} = \frac{\gamma}{\gamma-1} p \frac{dV}{d\theta} + \frac{1}{\gamma-1} V \frac{dp}{d\theta} + \frac{dQ_{loss}}{d\theta}, \quad Q_c = \int \frac{dQ_c}{d\theta} d\theta,$$

where Q_c is the heat generated from combustion, p is the cylinder pressure, V is the cylinder volume, γ is specific heat ratio, θ is crank angle, and Q_{loss} is the heat loss, respectively.

As can be seen, the heat release analysis requires a smooth continuous pressure trace and a precise heat loss model for an accurate result. In addition, computational burden may significantly increases due to following factors: to account for the dependency of specific heat ratio on temperature, cylinder temperature should be estimated in real-time; filtering of pressure trace may be required due to sensor noises; when sampling rate is too low, one may need to re-construct a continuous pressured trace from the sampled pressures; computational burden linearly increases as the number of cylinders increases.

To reduce computational burden, a novel control method is developed in this paper. The new method is based on principal component analysis (PCA) and requires a significantly less computational load and a lower sampling rate than a conventional method which performs heat release analysis. Besides, computational load of the new method is not proportional to the number of cylinders but is proportional to sampling rate. As such, the method is suitable for real-time combustion timing control [7, 8, 9] during transient as well as in steady state.

Principal Component Analysis

Principal component analysis (PCA) is a procedure to decompose a set of signals into a linearly uncorrelated signals called principal components through a linear transformation [10, 11, 12]. The PCA is also closely related to singular value decomposition (SVD) [13,14].

If the number of cylinders in an engine is M, and individual cylinder pressures are sampled N times per engine event, a matrix $P_{N \times M}$ can be formed from the sampled pressures. Then, a matrix of pressure deviations from ensemble average of the sampled pressures can be obtained as follows,

$$\Delta P_{N \times M} = P_{N \times M} - \mu_{N \times M},$$

where $\Delta P_{N\times M}$ is the matrix of pressure deviations, $\mu_{N\times M} = [\mu_N, \mu_N, \dots \mu_N]$, and μ_N is a vector of ensemble average of the sampled pressures, respectively.

Singular value decomposition of $\Delta P_{N\times M}$ results in

$$\Delta P_{N\times M} = U\Sigma V^T = U\eta,$$

where U is an N x N unitary matrix, Σ is an N x M non-negative rectangular diagonal matrix, and V is an M x M unitary matrix, respectively. The columns of U are normalized and orthogonal to each other, which are called principal modes. The rows of an N x M matrix η are called modal coefficients of a corresponding principal mode, respectively. Since U is a unitary matrix, the matrix η can be also obtained as follows if U is known,

$$\eta = U^T \times \Delta P_{N\times M}.$$

The columns of U are also the eigenvectors of the covariance matrix of $\Delta P_{N\times M}$ since

$$\Delta P_{N\times M} \times \left(\Delta P_{N\times M}\right)^T \times U = U\Sigma V^T \times V\Sigma^T U^T \times U = U\Sigma\Sigma^T = U\lambda,$$

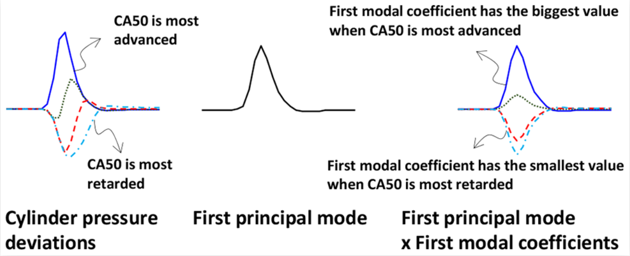
where λ is an N x N diagonal matrix with associated eigenvalues.

Property of First Modal Coefficients

According to SVD, the most significant principal mode is the first column of U and is called the first principal mode. The elements of the first row of η are the multiplication factors of the first principal mode for individual cylinder pressures and are called the first modal coefficients. In this section, property of the first modal coefficients and its application for combustion control are discussed with examples.

Figure 1 shows typical pressure deviations of individual cylinders from ensemble average of a four-cylinder engine when CA50s of individual cylinders are all substantially different. The figure also shows the first principal mode and that multiplied by its coefficients for individual cylinders, respectively.

FIGURE 1 Example of first modal analysis with individual cylinder pressure traces. The vector norm of a principal mode is 1.



In the figure, it is shown that when CA50 is more advanced, pressure deviation is more positive and as a result, the first modal coefficient must be bigger. In contrast, when CA50 is more retarded, pressure deviation is more negative, and the first modal coefficient must be smaller and even can be a negative number.

On the other hand, if all the cylinders are well balanced, it is expected that the first principal coefficients are all zero since pressure deviations from ensemble average are zero while the vector norm of the first principal mode is 1.

For verification, Figure 3 compares the first modal coefficients of an eight-cylinder engine when cylinders are all balanced with those when cylinders are imbalanced due to various CA50s, as depicted in Figure 2. Indeed, the figure shows that first modal coefficients are close to zero when cylinders are well balanced while first modal coefficients are inversely proportional to CA50 when cylinder are imbalanced. Thus, first modal coefficients of individual cylinder pressure traces can be used as an index to detect a cylinder imbalance, and further provide relative

FIGURE 2 Cylinder pressure traces used for principal component analysis.

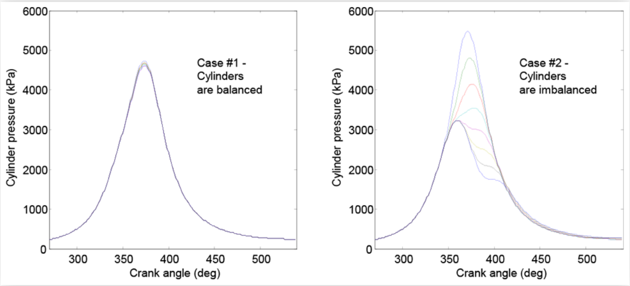
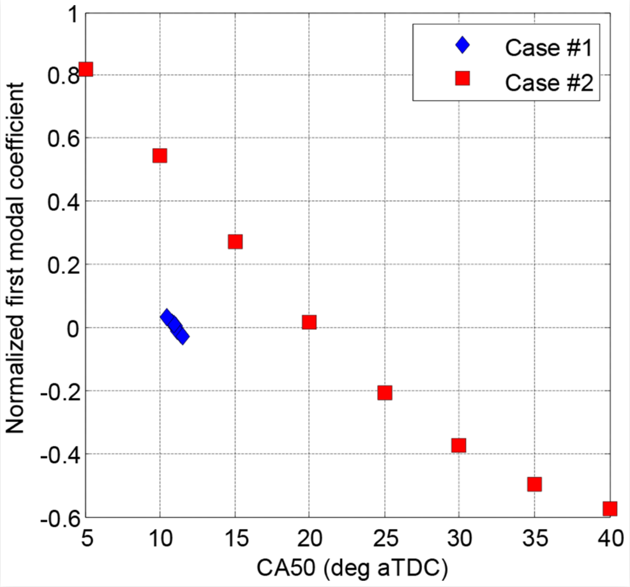


FIGURE 3 Comparison of first modal coefficients when cylinders are balanced versus imbalanced.



distribution of CA50s of individual cylinders if cylinders are imbalanced.

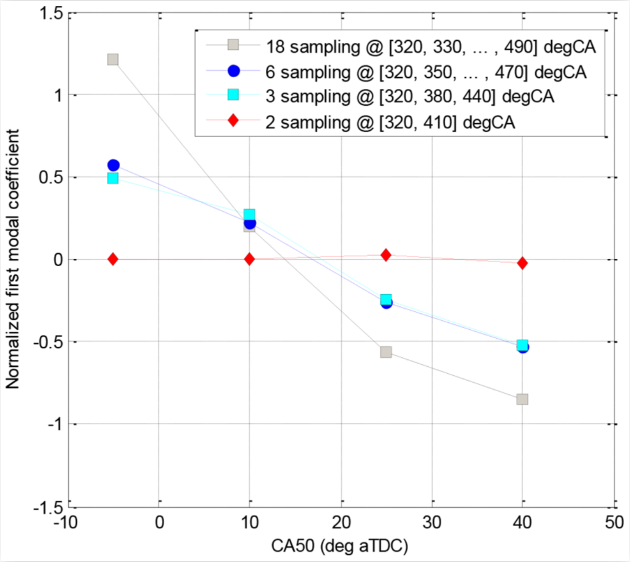
Computational Load

One of the main concerns with the conventional method to control CA50 is significant computational burden due to real-time heat release analysis performed at each engine event. In this section, computational load of PCA is discussed in detail.

The task that requires the heaviest computation in PCA is obtaining *U*, the eigenvectors of the covariance matrix of $\Delta P_{N \times M}$. However, it should be noted that the computational load of PCA is not proportional to the number of cylinders but proportional to the number of samplings of cylinder pressure trace since the size of the covariance matrix is *N* x *N*. In addition, PCA is performed once per engine cycle unlike the conventional method which performs heat release analysis once per engine event. Thus, it is expected that computational load of PCA is significantly less than that of the conventional method if the number of pressure samplings, *N*, is sufficiently small.

To find the minimum sampling requirement in PCA, first modal coefficients are obtained when the number of samplings per event is varied given a set of imbalanced cylinder pressure traces. The results are plotted in Figure 4. The figure shows that the first modal coefficients are no longer sensitive to CA50 when cylinder pressure traces are sampled twice per engine event. Thus, the minimum number of samplings per engine event is 3, and a major computational burden in PCA will be obtaining the eigenvectors of a 3 x 3 covariance matrix once per engine cycle.

FIGURE 4 First modal coefficients of a set of cylinder pressure traces when the number of samplings per event is varied.



Combustion Timing Control via Cylinder Balancing

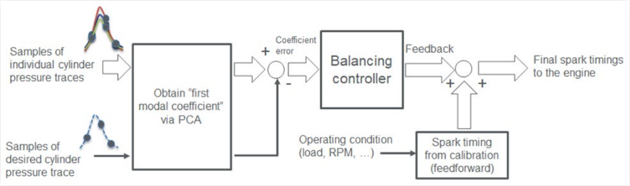
In this section, a combustion timing control strategy is developed using the property of first modal coefficients. As previously discussed, first modal coefficients of individual cylinder pressure traces can provide relative distribution of CA50s of individual cylinders if cylinders are imbalanced. If cylinders are all balanced, the first modal coefficients are zero.

The key idea of the proposed control strategy is to balance the individual cylinder pressure traces to a desired cylinder pressure trace using the property of first modal coefficients. Thus, the objective of combustion timing controller is to achieve the first modal coefficients of a set of the desired cylinder pressure trace plus the individual cylinder pressure traces to be zero all the time.

The desired cylinder pressure trace can be obtained from an engine operated in a test cell where the engine is calibrated to be operated in optimal condition. If a crank-angle based engine model is available, one may also obtain the desired pressure trace from simulation. Once the desired pressure trace is determined, it is sampled and stored as a function of, for example, engine speed and load such that the combustion timing control can be also seamlessly applied during transient conditions.

Figure 5 shows the schematic of the combustion timing control using first modal coefficients. A balancing controller is implemented to balance the cylinder pressure traces to the desired cylinder pressure trace by adjusting spark timing at each cylinder. The balancing controller determines the individual spark timing based on the deviations between the first modal coefficient of the desired cylinder pressure trace and that of the cylinder pressure traces using the property that first modal coefficients are inversely proportional to CA50 if cylinder pressure traces are not balanced. When all the cylinder pressure traces are balanced to the desired cylinder pressure trace, the first modal coefficients will be zero, and combustion timing of each cylinder will be equal to that of the desired cylinder pressure trace.

FIGURE 5 Balancing control strategy using first modal coefficients to control combustion timing of individual cylinders.



Validation

The combustion timing control in Figure 5 is validated through a simulation in this section. It is assumed that the engine is spark-ignited and has four cylinders. It is also assumed that the engine is operated with a constant engine speed and a fuel mass. For balancing, a PI controller is implemented at each cylinder to adjust spark timing, and a set of desired cylinder pressure traces is obtained from a high-fidelity crank-angle based engine model by setting CA50 to be 0 and 10 deg aTDC in crank angle, respectively.

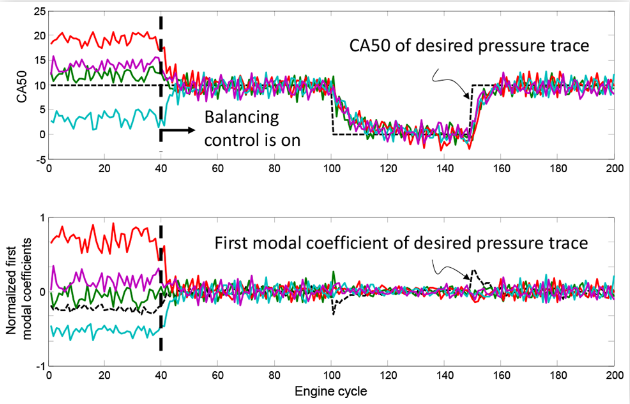
The crank-angle based engine model is also used to generate cylinder pressure trace at each engine event given a spark timing. Combustion timing is modeled to be proportional to spark timing, but a constant bias and a random noise are added to the onset of combustion at each cylinder.

Individual cylinder pressure traces are sampled three times per event at 320, 380, 440 degrees in crank angle, and PCA is performed at each engine cycle. The balancing controller are tuned on when engine cycle is equal to 40, and the simulation result is shown in Figure 6.

The figure shows that cylinders are imbalanced at the beginning as indicated by CA50s and non-zero first modal coefficients. Once the balancing controllers are turned on, however, individual spark timings are adjusted to balance pressure traces to the desired pressure trace based on first modal coefficients. In less than 10 engine cycles, it is seen that all the cylinders are balanced, and first modal coefficients converge to zero.

When engine cycle is equal to 100, the desired cylinder pressure trace is switched to the other. Nonetheless, it is seen that the combustion timings of individual cylinders are successfully managed to follow the new set point while first modal coefficients are tightly controlled to be zero.

FIGURE 6 Simulation result with combustion timing control.



Conclusions

Conventional method to manage combustion timings of an engine equipped with individual cylinder pressure transducers is to maintain CA50 to be the optimal value. To calculate the CA50, heat release analysis is performed at each engine event, which poses a significant computational burden.

A novel method to maintain the optimal CA50 is developed in this paper. The new method balances individual cylinder pressure traces to a desired cylinder pressure trace based on first modal coefficients. To obtain the first modal coefficients, principal component analysis is performed with a set of pressure traces sampled from individual cylinders and a desired cylinder pressure trace that achieves the optimal combustion.

The new method uses the following property of first modal coefficients for balancing, i) the first modal coefficients are zero if cylinder pressure traces are all balanced, ii) if cylinder pressure traces are imbalanced, the first modal coefficients are non-zero and inversely proportional to combustion timings.

Computational load of the proposed method is not proportional to the number of cylinders but proportional to the number of samplings of cylinder pressure trace. Also, since the principal component analysis is performed at each engine cycle, computational burden is significantly less than the conventional method which performs heat release analysis at each engine event.

The method was validated with a simulation. A high-fidelity crank-angle based engine model was used to generate individual cylinder pressure traces at each engine event, and the method is applied to balance the cylinder pressure traces. The result shows that the new combustion timing control balances cylinder pressure traces to the desired cylinder pressure trace and as a result, achieves the desired CA50.

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